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The Compendium*

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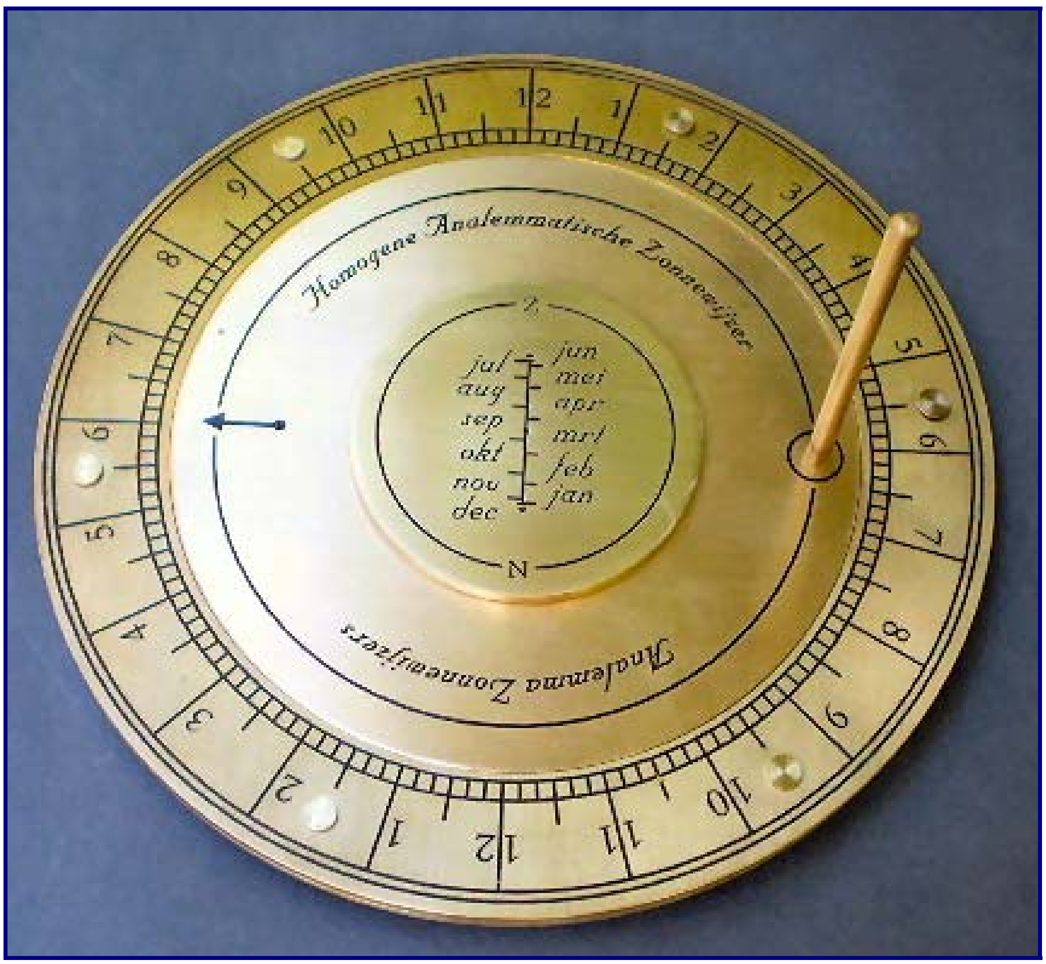

A Homogeneous Analemmatic Sundial by Hendrik Hollander

There is more day to dawn. - Henry David Thoreau

* *Compendium... "giving the sense and substance of the topic within small compass." In dialing, a compendium is a single instrument incorporating a variety of dial types and ancillary tools.*

Homogeneous Analemmatic Sundials

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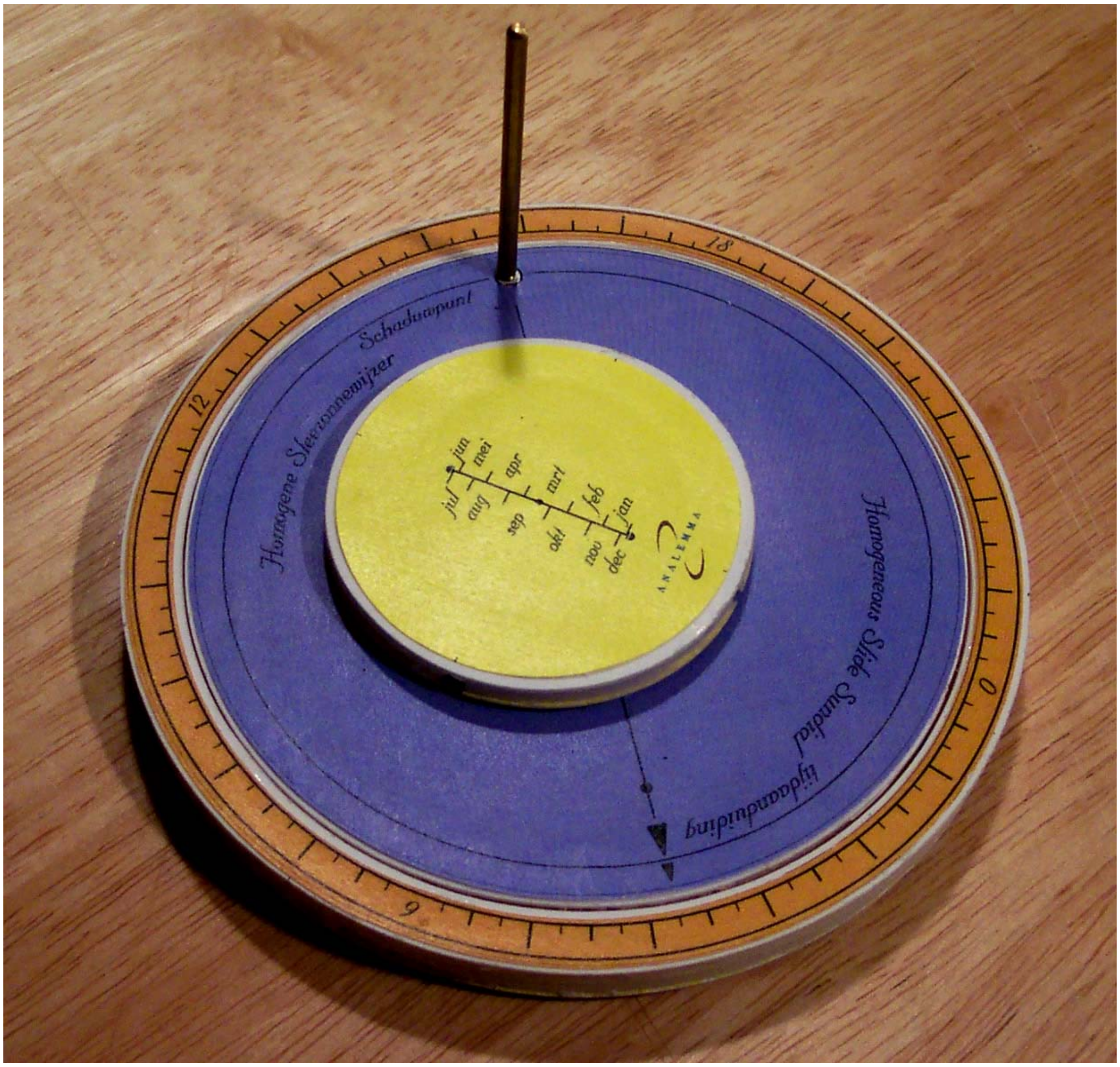


Sundials which have equally spread hour points on a circle are called "Homogeneous". Well known examples are the equatorial sundial and the Foster-Lambert Sundial. An advantage of the homogeneous sundial is its capability to adjust for the equation of time, the longitude or the daylight saving time in a simple way. The ring with the hour points is simply rotated the needed degrees. In this article a homogeneous sundial is described which has a shadow caster which moves during the day with respect to the circle with the hour points: a Homogeneous Analemmatic Sundial.

<u>The homogeneous analemmatic sundial with vertical gnomon</u>

To understand the homogeneous analemmatic sundial, we start from the standard analemmatic sundial for a latitude φ (for the figures in this article, φ equals 52º). The hours of the analemmatic sundial are found on an ellipse as shown in Figure 1. The ellipse can be constructed by using 2 circles with an homogeneous hour point distribution. The radius of these circles equals the half short axis and the half long axis of the ellipse.

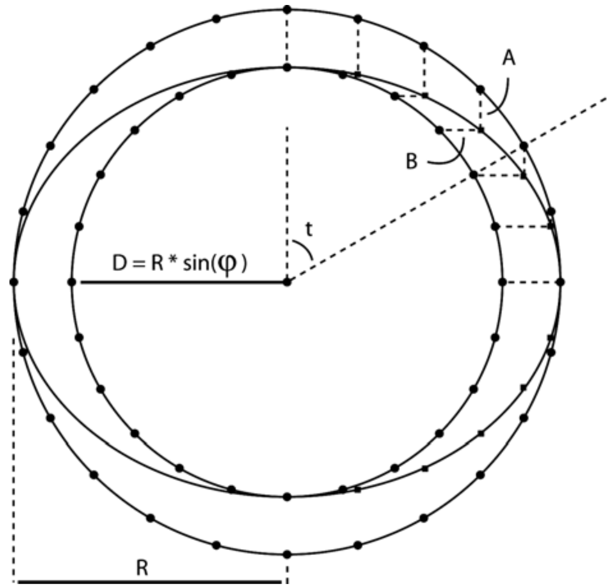


Figure 1: Cconstruction of an analemmatic sundial using 2 circles with an homogeneous hour point distribution

The ellipse is found by a translation of the homogeneous hour points of the greatest circle over a distance *A*, parallel to the short axis. The ellipse can also be found by a translation of *B* of the hour points of the smallest circle, parallel to the long axis of the ellipse. Defining *R* as the half long axis and *D* as the half short axis, we find $D = R * sin(\varphi)$. Further the hour angle *t* is shown and the hour points on the ellipse are (partly) shown as small rectangles.

By translation of the angle-point (of the hour angle) over a distance *A*, the dotted lines of the hour angle will intersect with the hour points on the ellipse. Around this angle-point we can draw the circle with homogeneous hour points, see Figure 2. The translation over *A* is dependent on the place of the hour points on the ellipse (and therefore dependent on the hour angle *t*). Notice that the translation over *B* has the same effect.

The mechanical construction of the device as described above is simple. Two trenches are placed perpendicular. Each trench holds a small pivot '*p*' and '*q*'. The pivots are connected by rod *K* of length *R*. The rod is split into parts *C* and *D*, so $R = D+C$, $D = R * sin\ \varphi$ and $C = R - R * sin\ \varphi$. The length *D* equals the short axis of the ellipse. By turning the rod *K* around, the pivots *p* and *q* will slide through the trenches and point *M* on the rod will mark the ellipse, see Figure 3.

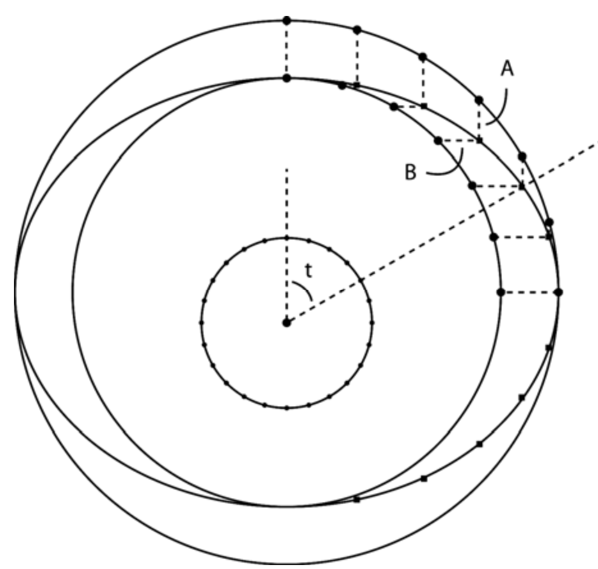


Figure 2: Translation of the angle-point over A leaving the homogeneous distribution untouched

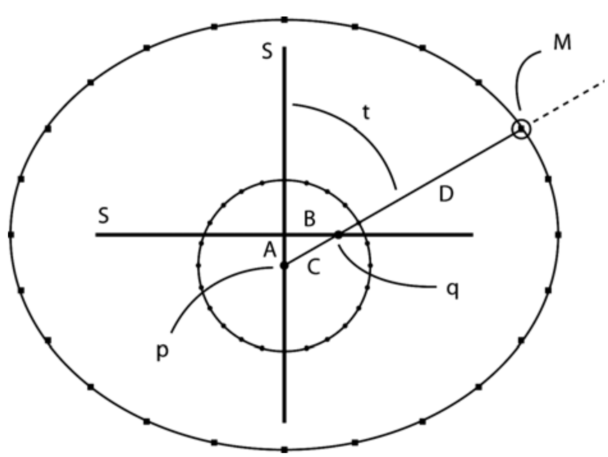


Figure 3: Mechanical construction of the ellipse with translation over A and B

The translations *A* and *B* as described in the previous figures are found in the center of the ellipse. Also the hour angle *t* is shown. The time can be read from a circle with homogeneous hour distribution with its center on pivot *p* (or *q*). The position of this circle differs during the day. The latitude *φ* for which the sundial is designed can simply be determined by the position of the pivot *q* on the rod *K*.

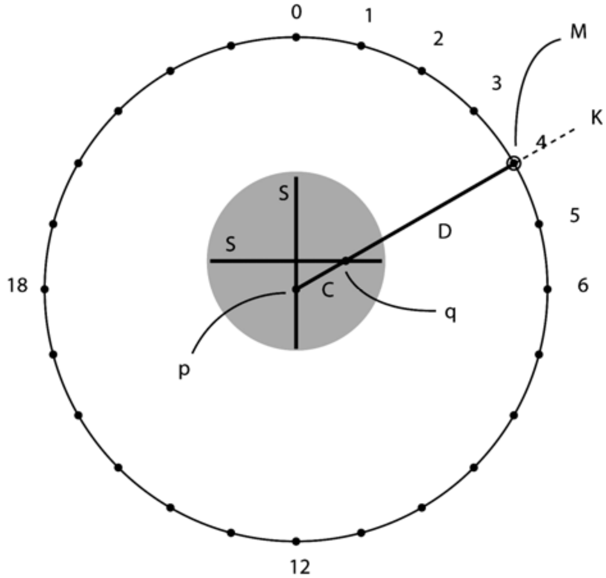


Figure 4: Homogeneous analemmatic sundial with gnomon, de gnomon is placed at M

By using the circle with the hour points as the fixed reference (and magnify it a bit), we find the homogeneous analemmatic sundial as shown in Figure 4. Pivot *p* is the center of the sundial. Pivot *q* moves the central disc up and down with respect to the circle with hour points. The gnomon is placed at point *M*.

The line with the loci of shadow points is drawn on the central disc as usual for the analemmatic sundial (see Figure 5). The shadow of the gnomon must intersect this line at the right date. The formula for the displacement (Δ) is found in the paragraph: "Formulae of the homogeneous analemmatic ".

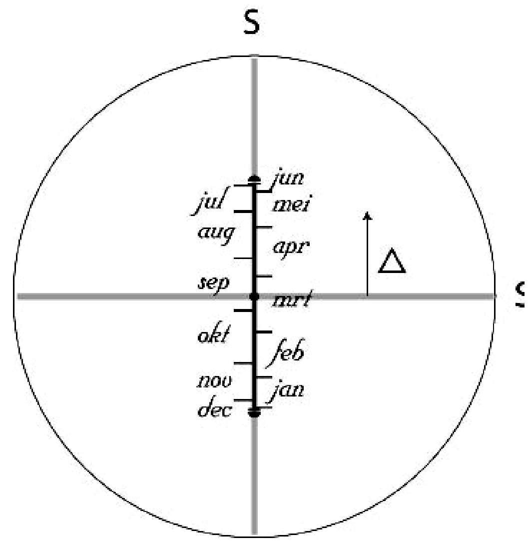
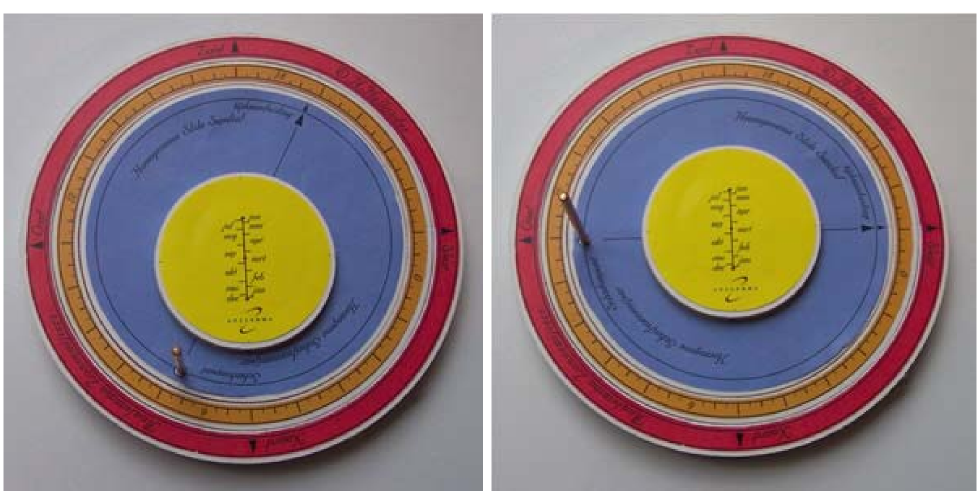


Figure 5: The central disc with the loci of shadow point for the different dates. The shadow of the gnomon must intersect with the right date

Notice that the sundial will mark the time by turning the rod K until the shadow of the gnomon will intersect the right date on the central disc. The north/south line equals the trench *S* which slides over pivot *p*, the center of the sundial. See Figure 6, the outermost (red) ring indicates the position to the north. The (orange) ring further to the center has the homogeneous hour points. Each hour equals (360/24=) 15 degrees. This ring can be rotated to adjust for the longitude or equation of time. The (blue) ring inside carries the vertical gnomon[1] of brass. The gnomon is placed on a distance *R* from the center of the sundial. This disc is rotated until the shadow of the sun intersects the right date on the central (yellow) disk. The central disk will move north and south while one adjusts the gnomon. The time is read from the ring with hour points, a small arrow is drawn opposite of the gnomon for this purpose[2].



Figures 6a,6b:

The homogeneous analemmatic sundial with vertical gnomon. While rotating the gnomon in its right place, the central disc will move north and south; these figures show 2 positions of the central disc

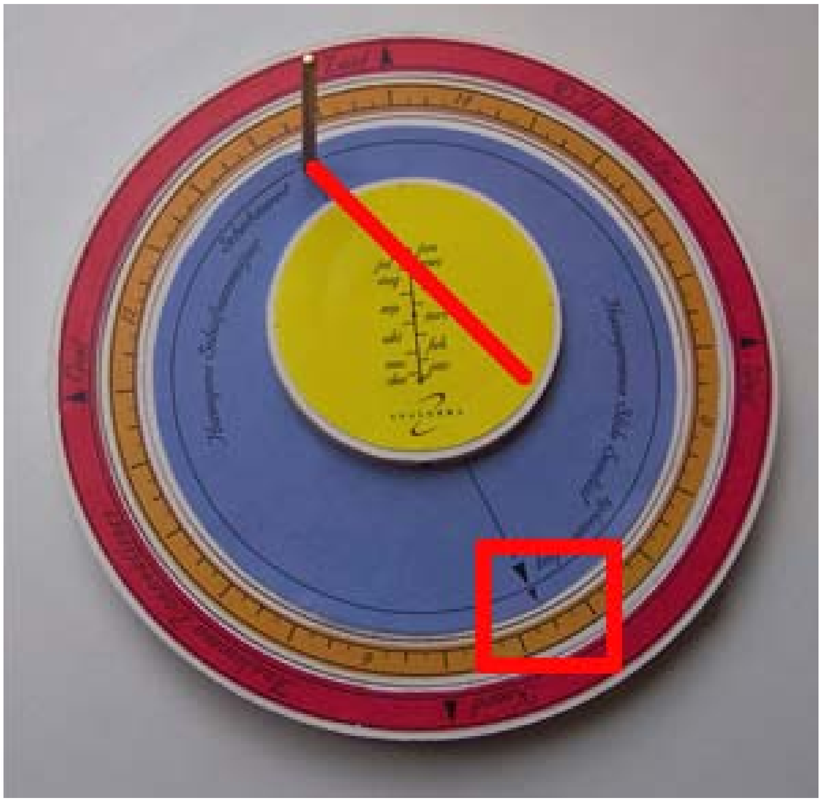


Figure 6c: The homogeneous analemmatic sundial with vertical gnomon. A possible shadow around June, the square marks the small arrow which marks the time.

The pictures in Figures 6a and 6b show 2 positions of the gnomon; the (yellow) central disc has moved due to the rotation of the gnomon. The picture in Figure 6c shows a possible shadow (around June); the square indicates the arrow which points out the time.

<u>The homogeneous analemmatic sundial with polar style</u>

The sundial as described above is based on an analemmatic sundial. The standard ellipse however is reshaped to a circle by the movement of the central disc. An analemmatic sundial is constructed by the projection of the equator and the polar style on the horizontal plane. See Figure 7. The short axis of the ellipse equals $2R * \sin \varphi$. If the equator is projected parallel to the polar style, we find the standard horizontal sundial with polar style. The equator will appear as an ellipse on the horizontal plane with a long axis which equals $2R / \sin \varphi$.

The ellipse of the horizontal sundial with polar style can also be reshaped to a circle by the use of the central disc. If we compare the ratio of the long axis and the short axis of the ellipses of the analemmatic sundial and the horizontal sundial with polar style, we find $\sin(\varphi)$ for both. For the analemmatic sundial, the short axis is north-south, for the horizontal sundial with polar style the short axis is east-west. So it is possible to turn the homogeneous analemmatic sundial with gnomon 90 degrees and mount a polar style on the central disc[3]. The central disc will move east-west. Doing so, we have created the homogeneous analemmatic sundial with polar style. The sundial will indicate the time when the shadow of the polar style intersects the point where previously the gnomon was mounted: the '<u>shadow point</u>'. Figure 8 shows 3 different positions of the central disc. The picture on the right shows the shadow point in the circle and the arrow which marks the time in the square.

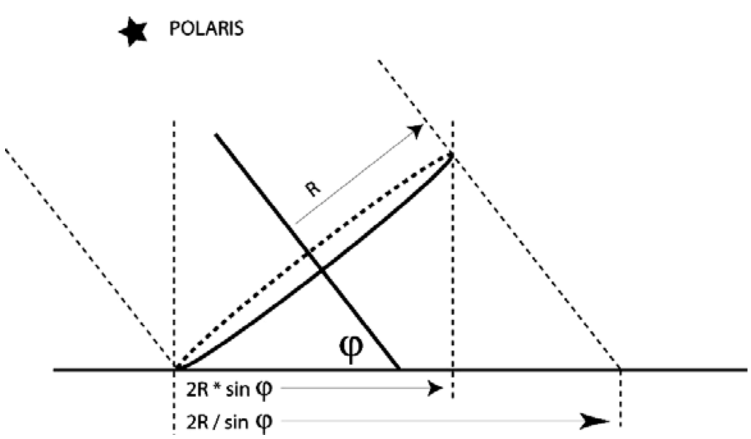


Figure 7: projections of the analemmatic sundial, perpendicular to the horizontal plane and parallel to the polar style

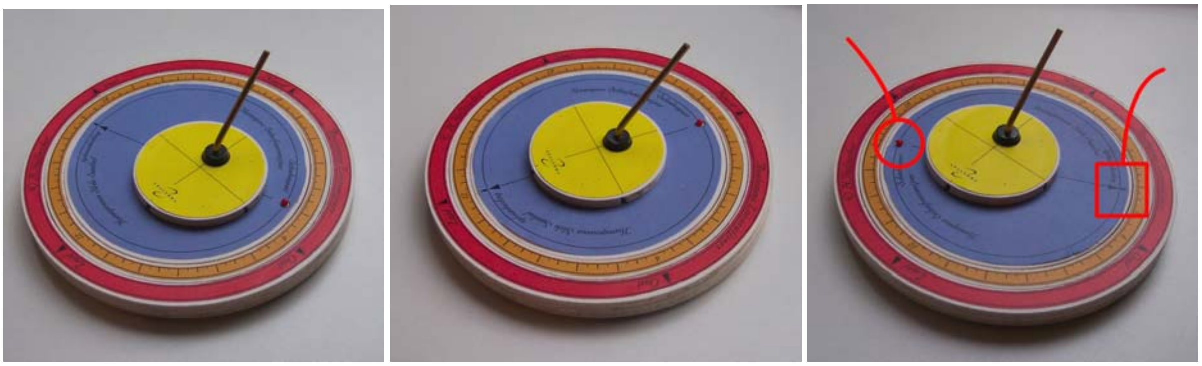


Figure 8: The homogeneous analemmatic sundial with polar style. The style points to the north as usual. Left and middle picture: 2 positions of the central disc. Right picture: the shadow point (in the circle) and the arrow which marks the time (in the square).

<u>The homogeneous analemmatic sundial with polar style marks the time by rotating the disc with the shadow point until the shadow of the polar style intersects with this shadow point.</u>

<u>The self aligning homogeneous analemmatic sundial</u>

By combining the gnomon and the polar style on the central disc, one can make a self aligning sundial, so a sundial which can be aligned the north be use of the shadow of the sun only. One has to choice whether the central disc will move north-south or east-west. To keep the gnomon out of the center of the sundial as much as possible, we choose for the north-south movement. This forces the sundial with the polar style to use the translation over *B* (see Figure 2). Therefore the shadow point will be closer to the center of the sundial. The shadow point is drawn on the sundial at a distance *D* from the center of the sundial (see Figure 9, the picture in the middle).

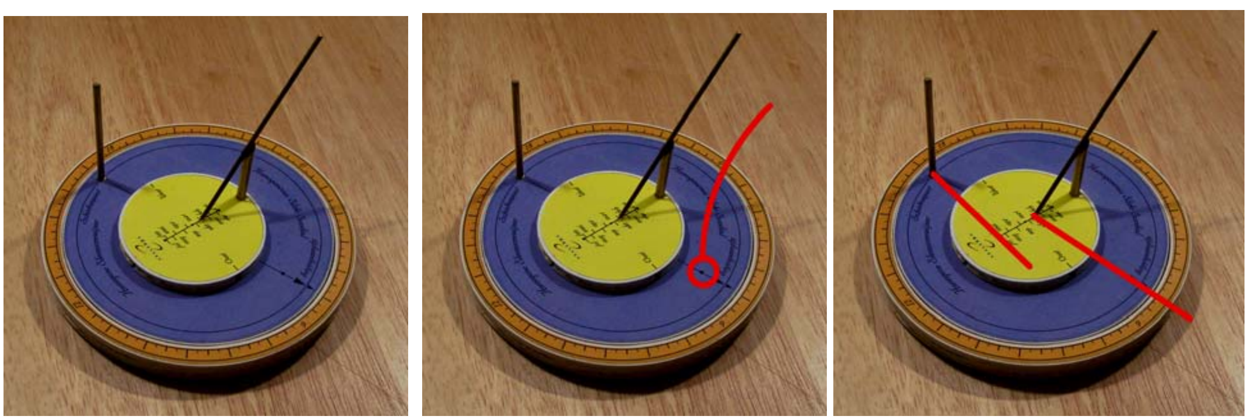


Figure 9: Self aligning homogeneous analemmatic sundial by combining the gnomon and the polar style. Left: the self aligning sundial. Middle: the shadow point for the polar style. Right: a possible shadow configuration (around June)

The sundial is turned until the shadow of the gnomon intersects with the right date on the central disc and the shadow of the polar style intersects with the shadow point. If this is done, the sundial will be aligned to the north.

<u>Adjusting the longitude correction</u>

The ring with the hour indication has to be rotated in the right position to compensate for the longitude and the equation of time. The longitude can be adjusted with the aid of a map of the world and a ring with the time zones. Figure 10 shows the back of the sundial (of Figure 9). The map is drawn in the center.

This map is connected to the north-south axis of the central disc on the upper side of the sundial. The time zones are drawn at the edge, this ring is connected to the ring with hour indication on the upper side of the sundial. By turning the ring with time zones with respect to the map, the longitude correction is adjusted.

Figure 10: The back of the sundial with the ability to adjust the longitude correction due to the map of the world and the time zones

Figure 11: the sundial of this picture will indicate MET in Utrecht (the center of the Netherlands)

The right longitude correction is found when one rotates the location where the sundial is used on the map, to the time zone one wants to read from the sundial. For example: Greenwich is turned to the GMT-time zone in Figure 10. Therefore, the sundial will indicate GMT-time in Greenwich. Also, Görlitz (Germany) is turned to the MET zone. Therefore, the sundial will indicate MET when used in Görlitz[4]. If one turns Utrecht (center of the Netherlands) to the MET-zone, the sundial will indicate MET when used in Utrecht, see Figure 11.[5] Besides the standard time zones, also some daylight saving time zones are present. If one turns the appropriate DST-time zone to the location where the sundial will be used, the day light saving time is read from the sundial.

<u>Categories of homogeneous sundials</u>

The proceeding types of homogeneous sundials can be distinguished as in the table below.

<u>Formulae of the homogeneous analemmatic sundials</u>

t = hour angle;
R = D+C (factor which determines the scale of the sundial)
$D = R * \sin \varphi$
$C = R - R * \sin \varphi$ (the distance between the pivots)
δ = declination of the sun
φ = latitude of the location where the sundial is designed for
A = translation, parallel to the short axis of the ellipse (east-west polar style, north-south for the gnomon)
B = translation, parallel to the long axis of the ellipse (east-west gnomon, north-south for the polar style)

<u>Homogeneous analemmatic sundial with gnomon</u>
loci of the shadow point on the central disc: $\Delta = R * \tan(\delta) * \cos(\varphi)$

in case of north-south translation of the central disc: A = C * cos (t)
the gnomon is placed at a distance R from the center of the sundial

in case of east-west translation: B = C * sin (t)
the gnomon is placed at a distance D from the center of the sundial

Homogeneous analemmatic sundial with polar style
the polar style is placed (as common) with an angle φ to the north on the central disc

in case of east-west translation of the central disc: A = C * sin (t)
the shadow point (the shadow of the style must intersect this point) is placed at a distance R from the center of the sundial

in case of north-south translation of the central disc: B = C * cos (t)
the shadow point (the shadow of the style must intersect this point) is placed at a distance D from the center of the sundial

Self aligning homogeneous analemmatic sundial

in case of north-south translation of he central disc:
A (related to the gnomon) = B (related to the polar style) = C * cos (t)
the gnomon is placed at a distance R from the center of the sundial

the polar style is place with an angle of φ to the north on the central disc
shadow point on a distance D from the center of the sundial

Table 1: Homogeneous Sundials

Yearly movement of the shadow points	Daily movement of the shadow caster	Self aligning	Type of sundial	Need to mount the shadow caster at different places through the year
Yes (line)	Yes	No	Homogeneous analemmatic sundial with gnomon	No
No (1 point)	Yes	No	Homogeneous analemmatic sundial with polar style	No
not applicable	No	No	Foster-Lambert sundial	Yes
not applicable	No	No	Equatorial sundial	No
Yes (gnomon) No (p. style)	Yes (both)	Yes	Self aligning homogeneous analemmatic sundial	No

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[1] In this article the gnomon is a vertical gnomon.
[2] One is free to choose a place for the arrow.
[3] The polar style intersects the horizontal plane which carries the shadow point, exactly at the crossing of the 2 trenches.
[4] The sundial indicates the local solar time in this position.
[5] The time zones like GMT and MET excluding the equation of time are read from the sundial in the described implementation.